

51.) $\frac{6}{y} + \frac{7}{y^3}$
 $\frac{6y^2}{y^3} + \frac{7}{y^3}$
 $\frac{6y^2+7}{y^3}$

52.) $\frac{13}{x-1} + \frac{14}{2x-2}$
 $\frac{26}{2x-2} + \frac{14}{2x-2}$
 $\frac{40}{2x-2}$
 $\frac{40}{2(x-1)}$
 $\frac{20}{x-1}$

53.) $\frac{1}{x} - \frac{2}{x^2}$
 $\frac{2x-4}{x^2}$
 $\frac{x-2}{x^2} \cdot \frac{x^2}{x^2}$
 $\frac{x-2}{x^2} \cdot \frac{x^5}{2(x-2)}$

$\frac{x-2}{x^2} \cdot \frac{x^5}{2(x-2)}$
 $\frac{x^5}{2x^2}$
 $\frac{x^3}{2}$

54.) $\frac{9}{x^2+5x+6} + \frac{12}{x+3}$
 $\frac{9}{(x+3)(x+2)} + \frac{12}{x+3} \cdot \frac{(x+2)}{(x+2)}$
 $\frac{9+12x+24}{(x+3)(x+2)}$
 $\frac{12x+33}{(x+3)(x+2)}$
 $\frac{3(4x+11)}{(x+3)(x+2)}$

59.) $\frac{x^{-1} + x^{-2}}{1 - x^{-2}}$
 $\frac{\frac{1}{x} + \frac{1}{x^2}}{1 - \frac{1}{x^2}}$
 $\frac{\frac{x}{x^2} + \frac{1}{x^2}}{\frac{x^2-1}{x^2}}$
 $\frac{x+1}{x^2} \cdot \frac{x^2}{x^2-1}$
 $\frac{1}{x-1}$

$\frac{x+1}{x^2} \cdot \frac{x^2}{x^2-1}$
 $\frac{x+1}{x^2} \cdot \frac{x^2}{(x+1)(x-1)}$
 $\frac{1}{x-1}$

3.) $f(x) = \frac{x^2}{2} + \frac{1}{x}$
 $\frac{x^3}{2x} + \frac{2}{2x}$
 $\frac{x^3+2}{2x}$
RATIONAL FUNCTION

4.) $\frac{x^4 + 3^x - x^2}{x^3 - 2}$
NOT A RATIONAL FUNCTION
 {Exponential}

5.) $f(x) = \frac{\sqrt{x+1}}{x+1}$
NOT A RATIONAL FUNCTION
 \sqrt{x} not a polynomial

6.) $f(x) = \frac{x^3}{2x^2} + \frac{1}{6}$
 $= \frac{x}{2} + \frac{1}{6}$
 $= \frac{3x}{6} + \frac{1}{6}$
 $f(x) = \frac{3x+1}{6}$
RATIONAL

8.) $\lim_{x \rightarrow \infty} (2x^{-3} + 4)$
 $= \frac{2}{x^3} + 4 = \frac{2}{\infty} + 4 = 0 + 4 = 4$

9.) $\lim_{x \rightarrow \infty} (3x^{-2} + 5x + 7)$
 $= \frac{3}{x^2} + 5x + 7 = \frac{3}{\infty} + \infty + 7 = 0 + \infty + 7 = \infty$

10.) $\lim_{x \rightarrow \infty} \frac{4x+3x^2}{4x^2+3x}$
 $= \frac{3x^2+4x}{4x^2+3x}$
 $\lim_{x \rightarrow \infty} \frac{3x^2}{4x^2} = \frac{3}{4}$

$$11.) \lim_{x \rightarrow -\infty} \frac{3x^2 + x}{2x^2 + 5x^3}$$

$$\lim_{x \rightarrow -\infty} \frac{3x^2}{5x^3} = \boxed{0}$$

$$12.) h(x) = 3 - \frac{1}{x} + \frac{x}{x+1}$$

$$\text{As } x \rightarrow \pm\infty \frac{1}{x} \rightarrow 0$$

$$\frac{x}{x+1} \rightarrow 1$$

$$\text{So } h(x) \rightarrow 3+1=4$$

$$\boxed{\text{HA: } y=4}$$

$$13.) f(x) = \frac{1}{1 + \frac{1}{x}}$$

$$\text{As } x \rightarrow \pm\infty \frac{1}{x} \rightarrow 0$$

$$\text{So } f(x) \rightarrow \frac{1}{1} = 1$$

$$\boxed{\text{HA: } y=1}$$

$$15.) f(x) = \frac{x^2+1}{x^2+5}$$

$$\boxed{x \rightarrow \pm\infty f(x) \rightarrow 1}$$

$$g(x) = \frac{x^3+1}{x^2+5}$$

$$x \rightarrow \pm\infty g(x) \rightarrow \frac{x^3}{x^2} = x$$

$$\boxed{\text{Graph approaches } y=x}$$

$$h(x) = \frac{x+1}{x^2+5}$$

$$x \rightarrow \pm\infty h(x) \rightarrow \frac{x}{x^2} = \frac{1}{x}$$

$$\boxed{\begin{array}{l} x \rightarrow \pm\infty h(x) \rightarrow 0 \\ x \rightarrow 0^+ h(x) \rightarrow \infty \\ x \rightarrow 0^- h(x) \rightarrow -\infty \end{array}}$$

17) Rational Function WITH:

$$\text{Even Symmetry: } f(x) = \frac{x^2}{x^2+1} \text{ or } f(x) = \frac{x^3}{4x^5}$$

Both Even or Both Odd

$$\text{Odd Symmetry: } f(x) = \frac{x^3}{x^2+1} \text{ or } f(x) = \frac{x^4}{3x^3}$$

One even and One Odd

$$\text{Neither: } f(x) = \frac{x^2+1}{x+1} \text{ or } f(x) = \frac{x+1}{x-1}$$

At least one neither

$$18.) r(x) = \frac{p(x)}{q(x)} \quad \begin{array}{l} p(x) \text{ degree } m \\ q(x) \text{ degree } n \end{array}$$

$$a.) \lim_{x \rightarrow \infty} r(x) = 0 \quad \boxed{n > m}$$

$$b.) \lim_{x \rightarrow \infty} r(x) = k \quad \boxed{n = m}$$

22) 2 Kg of bronze alloy that $\frac{1}{2}$ tin
Copper is constant adding x of tin
 $C(x) = \text{Concent of tin} = \frac{\text{total tin}}{\text{total alloy}}$

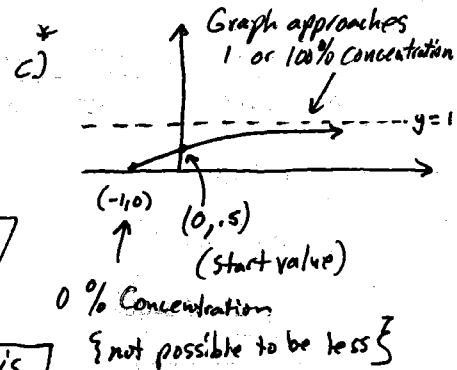
$$a.) C(x) = \frac{1+x}{2+x}$$

$$b.) C(.5) = \frac{1+.5}{2+.5} = \frac{1.5}{2.5}$$

60% alloy if .5 Kg tin is added

$$C(-.5) = \frac{1+(-.5)}{2+(-.5)} = \frac{.5}{1.5}$$

33.3% alloy is .5 Kg of tin is removed



0% Concentration
not possible to be less

25.) Alcohol solution 5 gal water, x gal alcohol
 $x > 0$. $f(x)$ is ratio volume alcohol to liquid.

$$a.) f(x) = \frac{x}{x+5}$$

ALC+WATER

The Concentration of alcohol containing 7g alcohol + 5g water

$$b.) f(7) = \frac{7}{7+5} = \frac{7}{12} = 58.333\%$$

$$c.) 0 = \frac{x}{x+5} \quad \boxed{x=0 \text{ The concentration is } 0\% \text{ when no alcohol is added.}}$$

$$d.) \text{HA: } y = \frac{x}{x+5} = 1 \quad \boxed{y=1 \text{ As the volume of alcohol increases, the concentration approaches } 1 \text{ or } 100\%.}$$

$$16.) f(x) = \frac{4-3x}{5x-4}$$

$$x = \frac{4-3y}{5y-4}$$

$$x(5y-4) = 4-3y$$

$$5xy-4x = 4-3y$$

$$5xy+3y = 4+4x$$

$$y(5x+3) = 4+4x$$

$$y = \frac{4+4x}{5x+3}$$

$$\boxed{f^{-1}(x) = \frac{4x+4}{5x+3}}$$